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REVIEW

Nonlinear Magnetohydrodynamics. By D. BISKAMP. Cambridge University Press, 1993, 378 pp. £50.

Magnetohydrodynamics is the natural extension of fluid dynamics to the dynamics of electrically conducting fluids permeated by a magnetic field, which may be in part externally applied (through currents in coils external to the fluid system) and in part due to currents flowing within the fluid itself. The subject has its origins in astrophysical developments of the inter-war years, and in the development of geomagnetic dynamo theory of the early post-war years, but explosive development of the subject only came in the late 1950's and 1960's with the development of thermonuclear fusion devices, and the intense interest in the possibility of extracting energy from controlled fusion processes. Theoretical work during that era was focused largely on problems of linear stability, since the problem of finding a stable magnetostatic equilibrium for a confined plasma presented the fundamental challenge. Textbooks of that era reflect this understandable preoccupation with linearizable problems, nonlinear problems being at that stage beyond the power of analytical or computational techniques.

The last two decades have seen a shift of emphasis, guided and provoked at every stage by the rapid development of computational technique and power, towards problems in magnetohydrodynamics that are essentially nonlinear in character. Typical among these problems are the nonlinear saturation of instabilities to which confined plasmas were found to be almost inevitably prone, the problem of fully developed turbulence, involving random velocity and magnetic fields, and problems of relaxation to nonlinear magnetostatic equilibria in which turbulent processes play an important part. Research monographs and textbooks covering this range of topics have been extremely rare, not only because the subject is intrinsically difficult, but also because many of the central problems continue to defy rational investigation.

In these circumstances, Dieter Biskamp's beautifully produced volume *Nonlinear Magnetohydrodynamics* is a remarkable contribution to the literature of the subject, which will help to rekindle interest in those central problems of magnetohydrodynamics that retain critical importance both in relation to the fusion problem, which provides much of the motivation for the selection of topics, and to problems in the magnetohydrodynamics of the solar atmosphere, which are particularly emphasized in the final chapter. Biskamp has provided a remarkable coverage of the subject, starting with the theory of magnetostatic equilibria and the theory of normal modes and instability (with emphasis on energy principles), moving on through consideration of the nonlinear evolution of MHD instabilities, the problem of magnetic reconnection when finite resistivity is taken into account, and the problem of MHD turbulence, and concluding with three chapters concerned with Tokamak plasmas, the dynamics of the reversed field pinch, and the magnetohydrodynamics of solar flares. The text rapidly develops its own momentum, helped by an outstanding quality of printing and layout, and it represents an achievement of outstanding quality, which will be required reading for any graduate student undertaking research in plasma or astrophysical magnetohydrodynamics.

One of the more controversial issues concerns the role of the magnetic helicity (the integral throughout the plasma domain of the scalar product of magnetic field and its vector potential). It was shown by Woltjer in 1958 that, under appropriate boundary conditions, and under the assumption of perfect conductivity (or equivalently zero magnetic resistivity), this quantity is invariant. More generally, however, as indicated by an argument given by Biskamp at the end of Section 2.2, the helicity within any closed magnetic surface (moving with the fluid) is also conserved, and in particular the helicity within any closed magnetic flux tube. It is generally understood that these flux tube helicities are no longer invariant when finite resistivity effects are taken into account; in fact, as shown in Section 6.7, it is precisely the non-zero rate of change of helicity which provides a pointer to the process of magnetic reconnection in the fluid, due to diffusion of field lines near neutral points. It is maintained, however, first on page 17, and repeatedly in later chapters, that despite this non-invariance of flux tube helicities, nevertheless the global helicity somehow survives 'as a robust invariant'. This indeed is the basis of J. B. Taylor's 1974 theory of relaxation to minimum energy states, in which it is conjectured that the only relevant invariant is the global helicity. The basis for this conjecture has never been adequately clarified, and I looked closely in this text to see whether any new light is shed on this important issue. Taylor's theory is first discussed in Section 7.3 (on self-organization and turbulence decay laws) and again in Chapter 9 in relation to the dynamics of the reversed field pinch. The justification for the assumption of invariant helicity (despite resistivity effects) remains however quite obscure. Certainly, the type of argument given at the foot of page 187 is fallacious. Here, it is stated that by simply looking at the equations for rate of change of energy E and magnetic helicity H, it may be seen that $\mathrm{d}H/\mathrm{d}t$ is small compared with $\mathrm{d}E/\mathrm{d}t$. The two quantities however have different physical dimensions, so the assertion is meaningless. What must be compared is $H^{-1}\mathrm{d}H/\mathrm{d}t$ and $E^{-1}\mathrm{d}E/\mathrm{d}t$, but this is not done. It is suggested that the magnetic helicity decays more slowly than the energy because there is an inverse cascade of helicity to large scales at which resistivity is much less effective. However, transfer of helicity to large scales may be accompanied by transfer of magnetic energy to large scales also (this after all is the mechanism of dynamo instability associated with the α-effect), so the argument appears inconclusive.

There is one circumstance when Taylor's conjecture must obviously be wrong. If all magnetic flux tube helicities are, say, positive (i.e. all magnetic linkages are right-handed), then the global helicity, being the sum of the flux tube helicities, decays at precisely the same rate as the individual flux tube helicities. In this circumstance, there can be no justification for assuming invariance of global helicity, while admitting non-invariance of flux tube helicities. Any argument that is devised by way of justification for the Taylor conjecture should be tested critically against this type of situation.

By contrast, if there are approximately as many right-handed linkages as left-handed linkages, then the rate of change of global helicity may well be much less than that of individual flux tube helicities; but now, the global helicity is itself small (being the sum of many positive and negative contributions) so that a proper estimate of $H^{-1}\mathrm{d}H/\mathrm{d}t$ is quite a delicate matter. I would like to have seen a more thorough discussion of this controversial issue, which is so central to the process that leads to equilibrium magnetostatic states.

This, however, is perhaps a small quibble in relation to a text that gives a comprehensive coverage of plasma dynamics, both in laminar and turbulent regimes. The selection of topics, the choice of illustrations, and the fluent discussion, all provide insight into a great range of phenomena. This text will do much to revitalize research in fundamental magnetohydrodynamics. As the first volume in the new series of Cambridge Monographs on Plasma Physics, it sets a very high standard; it is likely to be a standard reference on the subject for many years to come.

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